AP CALCULUS TEST PREP 1/17/09

NEW BRITAIN HIGH SCHOOL

DERIVATIVES OF LOGARITHMS AND EXPONENTS

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$$\frac{d}{dx}x^n =$$

$$\frac{d}{dx}e^x =$$

$$\frac{d}{dx}\ln x =$$

$$\frac{d}{dx}\log x =$$

$$\frac{d}{dx}a^x =$$

$$\frac{d}{dx}x^x =$$

2008 AB

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

   (a) Write an equation for the line tangent to the graph of f at $x = e^2$.

   (b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

   (c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

   (d) Find $\lim_{x \to 0^+} f(x)$.

Question 6

Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x\to 0^+} f(x)$.

(a) $f(e^2) = \dfrac{\ln e^2}{e^2} = \dfrac{2}{e^2}$, $f'(e^2) = \dfrac{1 - \ln e^2}{(e^2)^2} = -\dfrac{1}{e^4}$

An equation for the tangent line is $y = \dfrac{2}{e^2} - \dfrac{1}{e^4}\left(x - e^2\right)$.

$2 : \begin{cases} 1 : f(e^2) \text{ and } f'(e^2) \\ 1 : \text{answer} \end{cases}$

(b) $f'(x) = 0$ when $x = e$. The function f has a relative maximum at $x = e$ because $f'(x)$ changes from positive to negative at $x = e$.

$3 : \begin{cases} 1 : x = e \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$

(c) $f''(x) = \dfrac{-\frac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \dfrac{-3 + 2\ln x}{x^3}$ for all $x > 0$

$f''(x) = 0$ when $-3 + 2\ln x = 0$

$x = e^{3/2}$

The graph of f has a point of inflection at $x = e^{3/2}$ because $f''(x)$ changes sign at $x = e^{3/2}$.

$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer} \end{cases}$

(d) $\lim\limits_{x\to 0^+} \dfrac{\ln x}{x} = -\infty$ or Does Not Exist

1 : answer

2003 AB

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

36 The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

2003 AB

1. If $y = \left(x^3 + 1\right)^2$, then $\frac{dy}{dx} =$

(A) $\left(3x^2\right)^2$ (B) $2\left(x^3 + 1\right)$ (C) $2\left(3x^2 + 1\right)$ (D) $3x^2\left(x^3 + 1\right)$ (E) $6x^2\left(x^3 + 1\right)$

9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

1998 AB

2. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

AB–2 / BC–2 1998

2. Let f be the function given by $f(x) = 2xe^{2x}$.

   (a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

   (c) What is the range of f?

   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim_{x\to-\infty} 2xe^{2x} = 0$

$\lim_{x\to\infty} 2xe^{2x} = \infty$ or DNE

2 points:
- 1: 0 as $x \to -\infty$
- 1: ∞ or DNE as $x \to \infty$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
$f'(x) > 0$ for all $x > -1/2$

–or–

(ii) $f'(x)$: $-$ to the left of $-1/2$, $+$ to the right of $-1/2$

and $x = -1/2$ is the only critical number

3 points:
- 1: solves $f'(x) = 0$
- 1: evaluates f at student's critical point
  0/1 if not local minimum from student's derivative
- 1: justifies absolute minimum value
  0/1 for a local argument
  0/1 without explicit symbolic derivative

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

3 points:
- 1: sets $y' = be^{bx}(1 + bx) = 0$
- 1: solves student's $y' = 0$
- 1: evaluates y at a critical number and gets a value independent of b

Note: 0/3 if only considering specific values of b

AB–2 / BC–2 1998

Board Note # 1

Part (d)

3/3 Argument with the following three ingredients:

1. The graph of $y = bxe^{bx}$ is a horizontal compression or expansion (with a reflection across the y–axis if $b < 0$) of the graph of $y = xe^{x}$.
2. The range of $y = bxe^{bx}$ is therefore the same as the range of $y = xe^{x}$.
3. Therefore the absolute minimum value of $y = bxe^{bx}$ is the same for all (non–zero) values of b.

0/3 Analyzing the horizontal compression/expansion of graphs of $y = bxe^{bx}$ for specific values of b.

1998 AB

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim\limits_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

16. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $-\cos(e^{-x})$
(B) $\cos(e^{-x}) + e^{-x}$
(C) $\cos(e^{-x}) - e^{-x}$
(D) $e^{-x}\cos(e^{-x})$
(E) $-e^{-x}\cos(e^{-x})$

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258